Hydraulic characterisation of a karst aquifer using large-scale pumping tests in the conduit system

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Introduction - Use of numerical models for simulation of karst hydraulics

Prediction is an integral part for the sustainable management of groundwater resources and requires the development and application of mathematical models. This is especially true for karst aquifers, where groundwater flow is still considered hardly quantifiable. Groundwater flow in karst aquifers is focused in the highly conductive solution conduits. These, however, comprise only a small percentage of the total aquifer volume. The majority of the aquifer storage is typically provided by the fissured matrix.

In general, two different modelling approaches can be distinguished for the characterisation of karst systems (Kovács and Sauter 2007). Lumped parameter models do not take into account spatial heterogeneities within the aquifer. Thus, they cannot provide information on the aquifer geometry and the hydraulic parameter field. These models are usually not based on the physics of flow and therefore lack predictive power. In contrast, distributed parameter models are capable to simulate heterogeneities at any scale. However, input data for distributed parameter models are difficult and frequently impossible to obtain. That is because the development of a conduit system introduces extreme heterogeneities associated with a strong anisotropic character in the hydraulic parameter field of a karst aquifer. This leads to scaling issues, which have to be considered when parameterising mathematical models in order to simulate karst hydraulics. Király (1978) and Sauter (1992) demonstrated that the hydraulic conductivity of karst aquifers may change with the considered aquifer volume by several orders of magnitude. Small-scale borehole tests, for example, typically provide information about the hydraulic properties of the fissured matrix only. This is due to the fact that boreholes are usually located between karst conduits, and the radius of investigation is limited by the abstraction rate of the pumping well.

Teutsch and Sauter (1991) provided a classification scheme of different distributed modelling approaches in karst hydrogeology. The scheme combines the capability of a model to represent heterogeneities, their practical applicability, and the investigation effort required. The authors pointed out that combined discrete-continuum approaches (hybrid models) may be the most suitable mathematical models for karst hydraulic simulation, because this approach is capable to simulate the large-scale discontinuities existing in karst aquifers. The continuum of hybrid models may be used to simulate fissured matrix blocks, and can be parameterized by small-scale pumping tests. Consequently, the parameter identification problem is reduced to the detection of the geometry and hydraulics of the
conduit system. Artificial tracer tests may be a promising method to estimate conduit volumes between a sinkhole and a karst spring (Birk et al. 2005, Geyer et al. 2007). However, only limited information is gained about the geometry of a conduit system, and the exchange of the conduit system with the fissured matrix. Spring responses provide information about the entire karst aquifer, however, influenced by the respective recharge mechanism that has to be known for hydraulic characterisation.

Aim of the presented work, therefore, is to investigate how large-scale pumping tests (e.g. Maréchal et al. 2008) with considerable pumping rates of several hundred litres per second may be used for a scale-continuous characterisation of the hydraulic parameter field of karst aquifers. In a first approach we study the response of synthetic karst aquifers to the pumping process in order to determine the sensitivity of the system by variation of geometric and hydraulic parameters and the pumping rate.

**Modelling approach**

Synthetic karst systems are simulated by using the numerical hybrid model CAVE (Carbonate Aquifer Void Evolution; Birk et al. 2005, Liedl et al. 2003). The model couples a discrete pipe network, representing the conduit system of a karst aquifer, to the continuum flow model MODFLOW (McDonald and Harbaugh 1996), representing the fissured matrix. Flow within the pipes is calculated by the Darcy-Weisbach equation for turbulent flow conditions and the Hagen-Poiseuille equation for laminar flow. Volume conservation in the pipe network is implemented according to Kirchhoff's law. The transfer of water $\Gamma$ between the fissured matrix and the conduit network is considered by a linear exchange term:

$$ \Gamma = \alpha \cdot (h_m - h_c) $$

with $h_m$ as the hydraulic head in the fissured matrix, the hydraulic head in the pipe $h_c$, and the transfer coefficient $\alpha$. It should be noted that the model does not consider storage in the discrete pipe-network.

In order to compare the influence of different parameters on the model results we calculate the dimensionless sensitivity coefficient (Zheng and Bennett 2002):

$$ X_{i,k} = \frac{\partial \hat{y}_i / \hat{y}_i}{\partial a_k / a_k} $$

$\hat{y}_i$ ... model dependent value at the location $i$,

$a_k$ ... $k^{th}$ parameter whose impact on the model-dependent value is to be calculated.

Thus, the dimensionless sensitivity coefficient $X$ quantifies the change of the model-dependent value due to a parameter modification. A larger dimensionless sensitivity coefficient means that small parameter variations will cause an increased change of model-dependent values (e.g. hydraulic head).

**Model parameters for synthetic case studies**

The modelling studies were conducted with a view towards field investigations in Mediterranean karst groundwater systems located near Montpellier (South France). The synthetic karst systems of this study are represented by a rectangular domain of approximately 7 x 8 km² and possess a uniform thickness of 500 m (200 m above sea level (asl) to 300 m below sea level (bsl); Figure 1). The model domain is discretised in 100 m x 100 m horizontal grid cells. The discrete pipe network is located 30 m asl. To study the effect
of the conduit geometry on groundwater flow during pumping, we considered three different conduit systems, which may appear in nature (Dreybrodt 1988): (1) a single-pipe system, (2) a dendritic system, and (3) a network ("meshed system"); see Figure 2. For each of these scenarios, the volume of the discrete pipe network is 150 000 m³. The pipe diameters were added up to obtain the total length of the respective pipe network. For detailed information, see Figure 2. Additional hydraulic parameters for the fissured matrix and for the conduit system are derived from field studies concerning carbonate aquifers (Sauter 1992, Birk et al. 2005). Parameter values for the fissured matrix were specified as hydraulic conductivity $K_m = 2 \cdot 10^{-5}$ m·s$^{-1}$ and storage $S_m = 0.01$. Karst pipes were parameterized with a roughness $= 0.1$ m and a water transfer coefficient $\alpha = 0.001$ m$^2$·s$^{-1}$.

**Figure 1:** Scheme of the synthetic karst system.

Boundary conditions for flow are of the first kind (Dirichlet) in the south of the model domain with a constant hydraulic head of 76 m asl, and of the second kind (no flow) at the remaining lateral boundaries. The southern end of the conduit system represents a karst spring. It is directly connected to the Dirichlet boundary condition. However, to avoid negative discharge during pumping from the conduits, the boundary condition is flux-limited at this location (Bauer et al. 2003). If discharge is negative (i.e., flow from the spring into the conduit), the boundary condition switches from constant head (76 m asl) to constant flow (-0.01 m$^3$·s$^{-1}$). Steady-state recharge with a rate of 600 mm·year$^{-1}$ is assumed and 25% of the total recharge is directly injected into the pipe network. The remaining recharge is equally distributed over the whole model domain ("diffuse recharge"). For each scenario, a pumping well was set up within the central pipe at the same location. Pumping rates were varied between zero and 1.5 m$^3$·s$^{-1}$. The drawdown caused by pumping was evaluated at three observation wells in the pipe network and the continuum (Figure 2).
Steady-state simulation without pumping

Steady-state hydraulic heads without pumping are shown in Figure 3 for the three different pipe systems. While the strongly localized single-pipe system produces large hydraulic gradients within the continuum (fissured matrix), the spatially extended dendritic and meshed systems lead to a more uniform distribution of hydraulic heads. The water budget for the case studies reveals that the amount of water drained from the single-pipe system to the spring (0.649 m$^3$·s$^{-1}$) is much smaller than that of the dendritic (0.874 m$^3$·s$^{-1}$) and the meshed system (0.877 m$^3$·s$^{-1}$). The following relationship can be found: the larger the surface/volume ratio of the conduit system, the larger the amount of water drained from the pipe system and discharged at the karst spring. However, under the given boundary conditions the total discharge is the same for all scenarios. Consequently, the percentage of diffuse discharge from the model simulating a single pipe is much higher than that of the models with extended pipe systems.

![Figure 3: Distribution of hydraulic heads without pumping for different pipe systems.](image-url)
Simulation of pumping tests

The simulation of pumping tests is divided into three time periods. Period 1 represents the initial state and lasts one day. Period 2 considers the actual pumping process, i.e. water extraction from the pumping well and during 92 days. In different scenarios, the pumping rate was varied from 0.5 m$^3$.s$^{-1}$ to 1.5 m$^3$.s$^{-1}$ for each conduit geometry. Period 3 simulates the recovery curve and takes 92 days. The simulations demonstrate that the drawdown within the pipes is significantly higher than within the continuum, which refers to the large conductivity and the neglected storage of the pipe system (Figure 4). At low pumping rates the drawdown in the continuum is small. However, if the pumping rate exceeds the rate of spring discharge (obtained from the simulations without pumping) a significant drawdown in the continuum is simulated, too. The discharge of a karst spring can therefore be considered to be a critical threshold for the pumping process.

![Figure 4](image1.png)

**Figure 4:** Drawdown for observation well 1 (pipe) and observation well 3 (continuum) for two different pumping rates.

To compare the withdrawal of hydraulic heads for different conduit geometries, we calculated the dimensionless sensitivity coefficient for pumping rates ranging from 0.5 to 1.5 m$^3$.s$^{-1}$ for the simulated observation wells (Figure 5). Thus, the sensitivity is highest for the single-pipe system, whereas the dendritic and the meshed system show a similar behaviour with lower sensitivity. Furthermore, the sensitivity coefficient decreases with increasing distance of the observation well to the pipe network.

![Figure 5](image2.png)

**Figure 5:** Dimensionless sensitivity coefficient (z-axis) for different pumping rates.
Finally, the equipotential lines at the end of the pumping period depict the strong influence of the karst conduits on drawdown (Figure 6). Highly permeable conduit systems permit a rapid propagation of hydraulic stresses due to pumping. Thus, the drawdown of the extended pipe systems is distributed over a larger area of the model domain than that of the single-pipe system.

Figure 6: Hydraulic heads at the end of the pumping period (unit m).

Hydraulic parameter studies – Sensitivity analyses

Sensitivity analyses were conducted for all hydraulic parameters considered by the numerical model CAVE. Starting from a basic scenario, we varied different parameters. As basic scenario we used the above described models for all three conduit systems with a pumping rate of 1.0 m$^3$·s$^{-1}$. The ranges of parameter variation are given in Table 1.

Table 1: Hydraulic parameters used for simulations of synthetical karst systems.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Abbr. in Fig. 7/8</th>
<th>Basic scenario</th>
<th>Range for sensitivity analyses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage coefficient of the</td>
<td>Sm</td>
<td>0.010 [-]</td>
<td>0.005 [-] to 0.050 [-]</td>
</tr>
<tr>
<td>continuum $S_m$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydraulic conductivity matrix</td>
<td>Km</td>
<td>2·10$^{-5}$ m$^{-2}$·s$^{-1}$</td>
<td>1·10$^{-4}$ m$^{-2}$·s$^{-1}$ to 1·10$^{-6}$ m$^{-2}$·s$^{-1}$</td>
</tr>
<tr>
<td>$K_m$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water transfer coefficient $\alpha$</td>
<td>a</td>
<td>0.0010 m$^{-2}$·s$^{-1}$</td>
<td>0.0100 m$^{-2}$·s$^{-1}$ to 0.0001 m$^{-2}$·s$^{-1}$</td>
</tr>
<tr>
<td>Pipe roughness</td>
<td>roug</td>
<td>0.10 m</td>
<td>0.01 m to 0.50 m</td>
</tr>
<tr>
<td>Direct recharge</td>
<td>Rdir</td>
<td>25 %</td>
<td>5 % to 50 %</td>
</tr>
</tbody>
</table>

Dimensionless sensitivity coefficients were calculated for the three observation wells at the end of the pumping period. Figure 7 shows the sensitivity analyses with respect to hydraulic parameters of the continuum. The most sensitive parameters are the hydraulic conductivity of the fissured matrix as well as the storage coefficient. The largest impact on hydraulic heads is observed for reduced $K_m$. Associated sensitivity increases with increasing distance from karst conduits. A small matrix storage coefficient leads to an increased effect on hydraulic heads near conduits as well as within the karst pipes. Furthermore, sensitivity depends on the karst conduit network, e.g. matrix storage is most influential for the single-pipe system. In general, the single-pipe system shows the highest variability of parameter sensitivity.
The most influential parameter for the discrete pipe network is the water transfer coefficient $\alpha$ (Figure 8). The variation of $\alpha$ mainly affects the hydraulic heads within the karst pipes and in the continuum close to the pipe network. The hydraulic heads within our synthetic catchment seem to be insensitive to changes of other parameters like the percentage of direct recharge or pipe roughness.

Summary and Conclusions

In general, we found relative large differences between the localized single and extended conduit systems regarding draining characteristics as well as hydraulics during time periods with additional hydraulic stress, e.g. due to pumping. Therefore we used different pumping rates from zero to 1.5 m$^3\cdot$s$^{-1}$ and computed hydraulic heads in the fissured matrix and the pipes. We identified a critical threshold of the pumping rate, which is associated to the karst spring discharge under steady state conditions. After exceeding this threshold, significant drawdown was observed.

Subsequently we conducted a sensitivity analysis with respect to relevant parameters of the fissured matrix as well as of the conduit system. The analysis was performed for three synthetic catchments during a time period with water extraction from the karst pipes in the order of 1.0 m$^3\cdot$s$^{-1}$, i.e. above the mentioned threshold. Dimensionless sensitivity coefficients were used in order to compare the values for different parameters. We found that hydraulic parameters of the fissured matrix are more influential on the hydraulic heads in the matrix than conduit parameters. The most important parameter of the conduit system was the water transfer coefficient $\alpha$ controlling the exchange of water between conduits system and fissured matrix. Other conduit parameters appear to be of minor importance.

Hydraulic stresses due to direct water extraction from karst conduits are able to cause aquifer responses at great distance from the pumping well if the pumping rate exceeds the threshold set by the spring discharge. The so enforced spatial variation of the hydraulic heads can potentially be used to identify system-specific parameters, e.g. the pattern of the karst conduits or the water transfer coefficient. The development of a corresponding methodological concept for the scale-continuous aquifer characterization is subject of ongoing research.
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References


