To Study of the Appropriate Probability Distributions for Annual Maximum Series Using L-Moment Method in Arid and Semi-arid Regions of Iran

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ABSTRACT
In Probability distributions within hydrology different methods are used for their application. The most current of they have been central moment and with the using of computers, maximum likelihood method is used, too. This research was carried out in order to recognition of suitable probability with pervious common methods. In order to investigation of suitable probability distribution for annul maximum series (AMS), by using of L-moment method through hydrometric stations which exiting in region, were be selected 20 hydrometric stations for maximum discharges. According to results of this research for annual maximum discharge, P3 distributions and L-moment method, LP3 distribution and ordinary moment method, LN2 distribution and ordinary moment method, LP3 distribution and L-moment method, P3 distribution and ordinary moment method have been suitable distinguished for %40, %30, %20, %10 and %5 of stations, respectively. For three distributions (P3, LN2, and LN3) and ordinary moment method with %23 of stations with P3 and ordinary moment were selected.

Keywords: Linear moment, Maximum likelihood, Ordinary moment, Discharge, Frequency distribution function.

INTRODUCTION
Estimation of AMS is often required for watersheds with insufficient or nonexistent hydrometric information particularly in arid and semi-arid regions. Because parametric methods require a number of assumptions, nonparametric methods have been investigated as alternative methods. L-moment diagrams and associated goodness-of-fit procedures (Wallis, 1988; Hosking, 1990; Chowdhury et al., 1991; Pearson et al., 1991; Pearson, 1992: Vogel et al., 1993) have been advocated for evaluating the suitability of selecting various distributional alternatives for modeling flows in a region. For example, Wallis (1988) found an L-moment diagram useful for rejecting. Jain and Singh's (1987) conclusion that annual maximum flood flows at 44 sites were well approximated by a Gumbel distribution and for suggesting a GEV distribution instead. Vogel et al. (1993) used L-moment diagrams to show that the two- and three-parameter log-normal models (LN2 and LN3), the LP3 and the GEV distributions were all acceptable models of flood flows in the southwestern United States. Gholami et al. (2001) used L-moment diagrams to show that the Gumbel distribution was acceptable model for AMS series in the north of Iran. In the following sections; we employ L-moment diagrams to annual maximum series (AMS) data in Iran. The objective of the present study is to introduce and evaluate

the suitable probability distributions for modeling annual maximum series in arid and semi-arid regions of Iran.

Study Area

Regarding the climate conditions and average precipitation of 250 mm, Iran is considered as an arid and semi – arid region of the world, which is mostly encountered, with lake of water consequently leading to water, explore operations. Most of the rivers in arid regions of Iran are seasonal and their floods maybe come unavailable during a short time of rainfall seasons, and because of some special geological problems of this region, most of the permanent rivers contain saline water and are useless.

AMS data employed in this study include 20 gauging stations with stream flow record lengths of 20 or more years for better water resources management in these regions of Iran. These stations are located in Iran Central plateau as shown in Fig. 1. Overall, Iran is divided into 6 drainage basins.

Fig.1. Location of Study Area

MATHERIALS & METHODS

Methods

In order to recognition of suitable probability distributions for annual maximum series by using of suitable methods through hydrometric stations which existing in region were be selected 20 stations for analysis. After selection of suitable stations and taking of consideration flows from selection stations 25 years statistical period from water year 1971-72 through 1997-98, was selected as statistical similar period. Statistical missing of selection stations were is rebuilt by regression between stations by using of SPSS software. The values of residual sum of squares (RSS) by ordinary moment method obtained for used distributions and compared with values RSS for L-moment method and with respect to the lowest RSS for each distributions by two methods of ordinary moment and L-moment were be selected suitable distributions for each stations. The best of probability distributions applied to estimate T-year AMS.

L-moment analysis

L-moments are linear combinations of order statistics which are robust to outliers and virtually unbiased for small samples, making them suitable for flood frequency analysis, including identification of distribution and parameter estimation (Hosking, 1990; Hosking and Wallis, 1993). L-moments are defined as linear combinations of probability weighted moments (PWM):

$$\beta_r = E\{X[F_X(x)^r]\} \quad (1)$$

Where β_r is the *r*th order PWM and *Fx(x)* is the cdf of X. When r = 0, β_0 is the mean streamflow. Hence a sample estimate of the first PWM, which we term b_0 is simply the sample mean. Nevertheless, unbiased estimators are often preferred in goodness-of-fit evaluations such as L-moment diagrams. Unbiased sample estimates of the PWMs, for any distribution can be computed from

$$b = \frac{1}{n}\sum_{j=1}^{n} x_j \quad (2a)$$

$$b_1 = \sum_{j=1}^{n-1}\left[\frac{(n-j)}{n(n-1)}\right]x_{(j)} \quad (2b)$$

$$b_2 = \sum_{j=1}^{n-2}\left[\frac{(n-j)(n-j-1)}{n(n-1)(n-2)}\right]x_{(j)} \quad (2c)$$

$$b_3 = \sum_{j=1}^{n-2}\left[\frac{(n-j)(n-j-1)(n-j-2)}{n(n-1)(n-2)(n-3)}\right]x_{(j)} \quad (2d)$$

where x_j represents the ordered streamflows with x_1 being the largest observation and x_n the smallest. The PWM estimators in eqn. (2) can be more generally described using

$$b_r = \frac{1}{n}\sum_{j=1}^{n-r}\left[\frac{\binom{n-j}{r}}{\binom{n-1}{r}}\right]x_{(j)} \tag{3}$$

For any distribution, the first four L-moments are easily computed from the PWMs using

$$\lambda_1 = \beta_0 \tag{4a}$$

$$\lambda_2 = 2\beta_1 - \beta_0 \tag{4b}$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 - \beta_0 \tag{4c}$$

$$\lambda_4 = 30\beta_2 - 12\beta_1 - \beta_0 \tag{4d}$$

L-moment ratio diagrams

Analogous to the product moment ratios; coefficient of variation $C_v = \sigma/\mu$, skewness γ and kurtosis κ, Hosking (1990) defines the L-moment ratios

$$\tau_2 = \frac{\lambda_2}{\lambda_1} \equiv L - coefficient of\ variation \tag{5a}$$

$$\tau_3 = \frac{\lambda_3}{\lambda_2} \equiv L - skewness \tag{5b}$$

$$\tau_4 = \frac{\lambda_4}{\lambda_2} \equiv L - kurtosis \tag{5c}$$

where λ_r, $r = 1, ..., 4$ are the first four L-moments and τ_2, τ_3 and τ_4 are the L-coefficient of variation (L-C_v), L-skewness and L-kurtosis, respectively. The first L-moment λ_1 is equal to the mean streamflow μ, hence it is a measure of location. Hosking (1990) shows that λ_2, τ_3 and τ_4 can be thought of as measures of a distribution's scale, skewness and kurtosis, respectively, analogous to the ordinary moments σ, γ and k respectively.

RESULTS

According to results of this research for annual maximum discharge, P3 distributions and L-moment method, LP3 distribution and ordinary moment method, LN2 distribution and ordinary moment method, LP3 distribution and L-moment method, P3 distribution and ordinary moment method have been suitable distinguished for %40, %30, %20, %10 and %5 of stations, respectively. For three distributions (P3, LN2, and LN3) and ordinary moment method with %23 of stations with P3 and ordinary moment were selected.

Table1. Suitable Probability Distributions and Methods for AMS

Station	RSS	Suitable Probability Distribution	Suitable Method
Ghamsar	1.5	P3	L-moment
Yalfan	4.9	LP3	Ordinary moment
Bandabassi	1.32	LP3	L-moment
Abgarm	4.3	P3	Ordinary moment
Sira	2.1	P3	L-moment
Roodak	1.02	P3	L-moment
Dodahak	4.9	LN2	Ordinary moment
Ghale	1.9	P3	L-moment
Eskandari	0.04	P3	L-moment
Chamriz	0.25	P3	L-moment
Dashtbal	8.3	LN3	Ordinary moment
Polkhan	0.3	P3	L-moment
Safarzade	0.4	P3	L-moment
Daman	0.2	P3	L-moment
Jirofto	0.15	P3	L-moment
Ghariatolarab	8.1	P3	L-moment
Godarzarch	1.03	P3	L-moment
Bonkooh	2.8	P3	L-moment
Arie	2.3	P3	L-moment
Senobar	3.9	LN3	Ordinary moment

Tabale2. Estimated T-Years Annual Maximum Series

Station	T-Years					
	2	5	10	20	50	100
Ghamsar	0.7	1	1.2	1.3	1.5	1.6
Yalfan	16	21	23	25	27	28
Bandabassi	100	120	134	145	160	169
Abgarm	14	15	15.4	15.7	16	16.2
Sira	67	84	91	97	105	110
Roodak	7.7	10.4	11.8	12	14	14.1
Dodahak	50	65	79	80	90	96
Ghale	224	294	335	348	413	442
Eskandari	23	33	39	44	53	59
Chamriz	240	334	388	427	487	522
Dashtbal	28	48	63	68	95	109
Polkhan	141	212	257	271	350	387
Safarzade	160	225	280	320	420	450
Daman	179	255	294	321	361	384
Jirofto	10	13	15	16	18	18.9
Ghariatolarab	7.8	10.9	12.5	13.6	15.2	16.2
Godarzarch	22	41	59	76	113	145
Bonkooh	27	52	73	93	130	159
Arie	7.9	10.6	12	12.2	14.2	14.3
Senobar	13	19	22	25	28.5	30

CONCLUSIONS

The main objective of this study was to select a set of suitable probability distributions for modeling annual maximum series in arid and semi-arid regions of Iran. L-moment diagrams revealed that the Pearson type 3 (P3), log Pearson type 3 (LP3), three-parameter log-normal (LN3) distributions all provide acceptable approximations to the distribution of AMS in arid and semi-arid regions of Iran. Of all the models evaluated, the P3 distributions probably provide the best description of the distribution of AMS across the entire central plateau of Iran, however, separation of central plateau of Iran into broad homogeneous regions can improve our ability to discriminate among potential AMS frequency models such that the P3 distribution provides the best approximation to the distribution of AMS in the most of this region. Of the models tested, the LP3, LN2, P3 and Ln3 distributions provides the best approximation to the distribution of AMS using ordinary moment method.

Finally, we concluded that L-moment method is suitable to determine AMS probability distributions in the central plateau of Iran and P3 is the best probability distribution for modeling annual maximum series in arid and semi-arid regions of Iran.

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